

a continuous distribution. It is shown that linear error simply moves the lobe, while quadratic error reduces and broadens it symmetrically. Cubic error, the type resulting from our tabular third difference, makes the lobe nonsymmetric.

We feel that this effect may be useful in arrays having multiwavelength element spacings. Grating lobes are present, but their modified shapes make them recognizable as such; also their direction of slope tends to point out the main lobe.

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The Reflected Voltage of a Line with an Antenna Load for Pulse Excitation*

Let $e^+(t)$ be the forward-travelling voltage wave on a transmission line of characteristic impedance R_c as it appears across the load of impedance $Z(f)$, let $e^-(t)$ be the reflected wave at the same point, and let $E^+(f)$ and $E^-(f)$ be their respective Fourier transforms. These transforms are related by

$$E^-(f) = \left[\frac{Z(f) - R_c}{Z(f) + R_c} \right] E^+(f),$$

since the expression in brackets is the voltage reflection coefficient. Hence, the time function that describes the reflected wave is obtained by taking the inverse Fourier transform of $E^-(f)$:

$$\begin{aligned} e^-(t) &= \int_{-\infty}^{\infty} E^-(f) e^{i2\pi f t} df \\ &= \int_{-\infty}^{\infty} \frac{Z(f) - R_c}{Z(f) + R_c} E^+(f) e^{i2\pi f t} df. \end{aligned} \quad (1)$$

This expression gives the reflected wave as it appears across the load for any instant of time t in terms of the transform $E^+(f)$ of the incident wave $e^+(t)$ and the impedances $Z(f)$ and R_c .

Let us consider the incident wave

$$e^+(t) = A \frac{\sin 2\pi f_c t}{2\pi f_c t},$$

whose transform has the simple form

$$\begin{aligned} E^+(f) &= A/(2f_c) \quad \text{for } |f| < f_c \\ &= 0 \quad \text{for } |f| > f_c. \end{aligned}$$

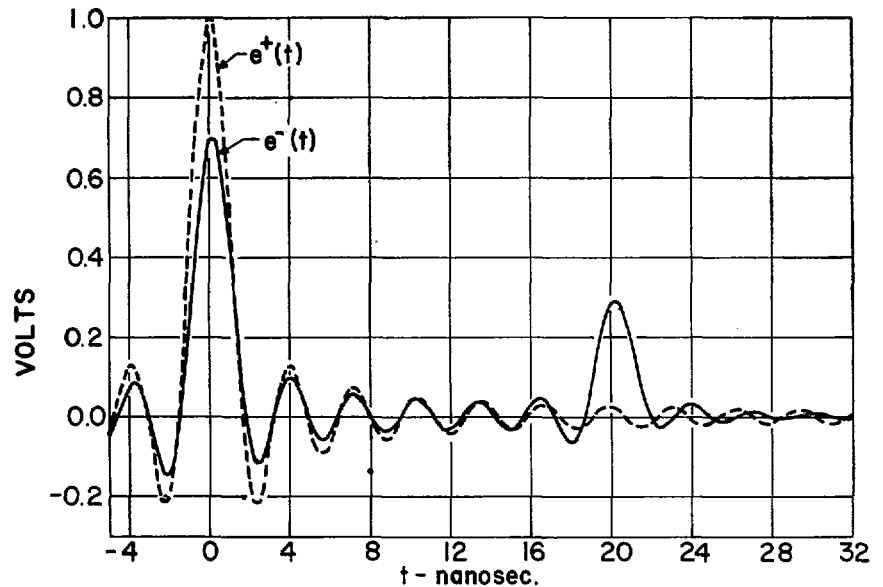


Fig. 1—Reflected voltage wave $e^-(t)$ for $e^+(t) = (\sin 2\pi f_c t)/(2\pi f_c t)$, where $f_c = 3.15 \times 10^8$ cps, at the load (consisting of a monopole of length $h = 3$ m and radius $r = 3.32$ mm) on a line of characteristic impedance $R_c = 50$ ohms. [Both $e^+(t)$ and $e^-(t)$ have significant nonzero values for $t < 5 \times 10^{-9}$ sec.]

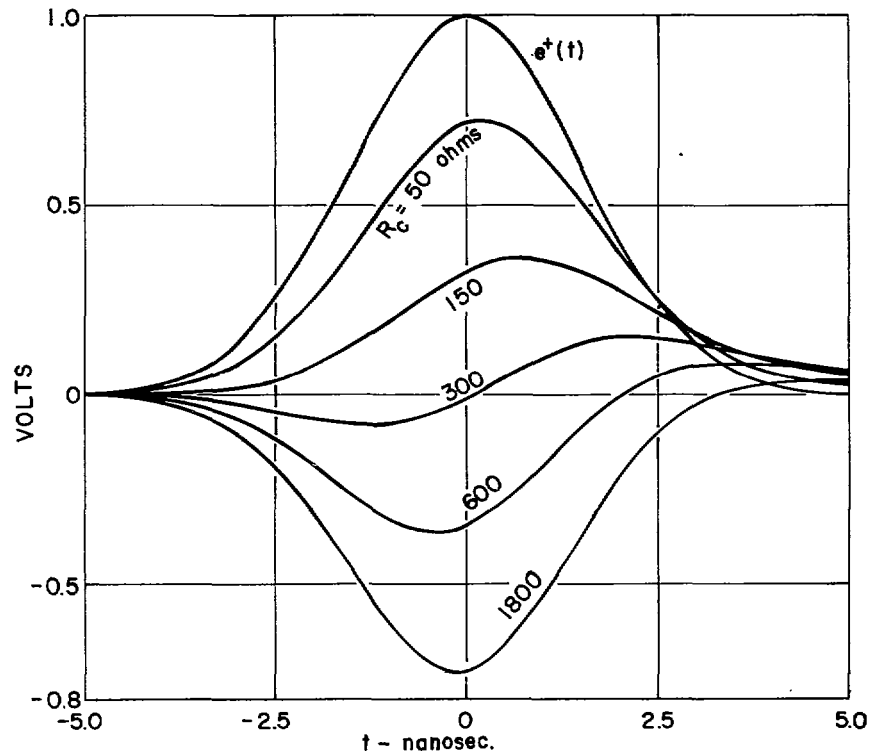


Fig. 2—Reflected voltage waves $e^-(t)$ for $e^+(t) = e^{-t^2/2t_1^2}$, where $t_1 = 1.5 \times 10^{-9}$ sec, at the load (consisting of a monopole of length $h = 3$ m and radius $r = 3.32$ mm) on lines of different values of characteristic impedance R_c . For $R_c = 0$, $e^-(t) = e^+(t)$; for $R_c = \infty$, $e^-(t) = -e^+(t)$. (Both $e^+(t)$ and $e^-(t)$ are less than 0.002 for $t < 5 \times 10^{-9}$ sec.)

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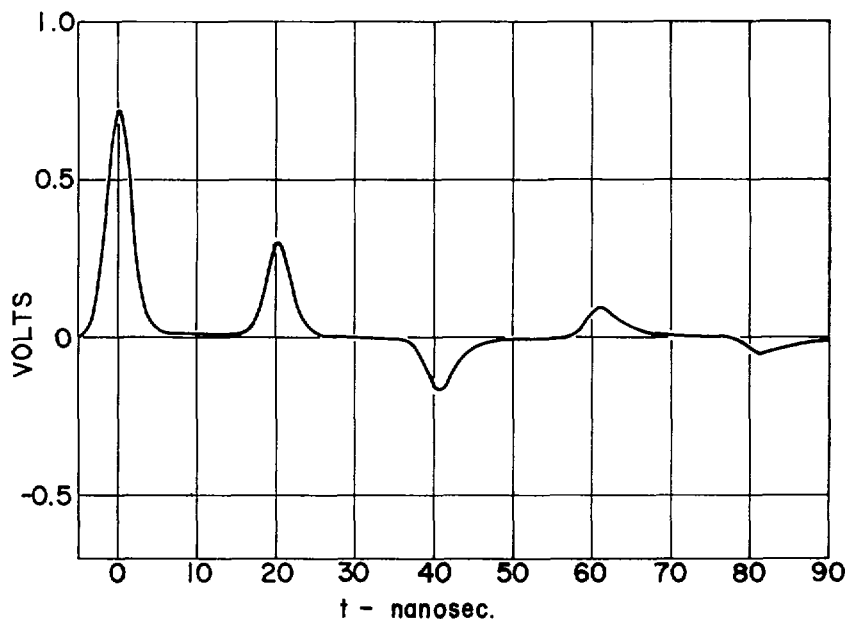


Fig. 3—Reflected voltage wave $e^-(t)$ of Fig. 2 for $R_c = 50$ ohms over an extended time range.

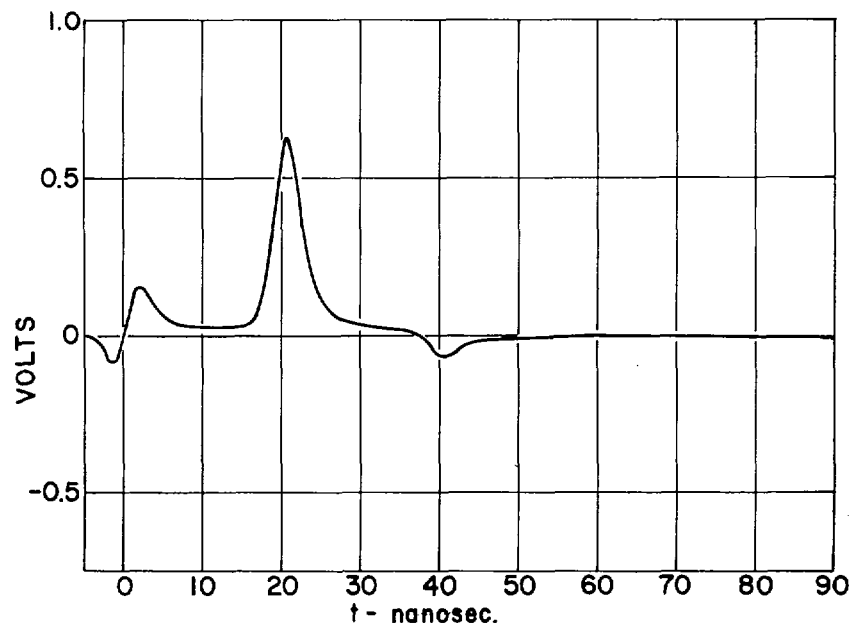


Fig. 4—Reflected voltage wave $e^-(t)$ of Fig. 2 for $R_c = 300$ ohms over an extended time range.

The reflected wave for this case is, from (1),

$$e^-(t) = \frac{A}{2f_c} \int_{-f_c}^{f_c} \frac{Z(f) - R_c}{Z(f) + R_c} e^{j2\pi f t} df. \quad (2)$$

We have integrated (2) numerically on Sandia Corporation's CDC-1604 computer for the case where $f_c = 3.15 \times 10^8$ cps, $A = 1$, $Z(f)$ is the impedance of a monopole of length $h = 3$ m and of radius $r = 3.32$ mm,¹ and $R_c = 50$ ohms. Fig. 1 shows both the incident wave and the reflected wave over a time interval of sufficient length to include the reflection from the junction as well as the first reflection from the end of the antenna.

As another example, we have evaluated (1) for the Gaussian incident pulse

$$e^+(t) = e^{-t^2/2t_1^2}, \quad t_1 = 1.5 \times 10^{-9} \text{ sec},$$

which has the transform

$$E^+(f) = \sqrt{2\pi} t_1 e^{-f^2/2t_1^2}, \quad f = 1/(2\pi t_1) \text{ cps},$$

for the same monopole antenna described above. The incident wave and the reflected wave for several values of R_c are shown in Fig. 2 over a time interval that includes only the reflection from the junction. Fig. 3 shows the reflected wave for the case $R_c = 50$ ohms over a more extended time range. Fig. 4 shows the case $R_c = 300$ ohms over the same extended range. For this latter case, a sort of "matched" condition holds with the result that the only large reflection comes from the end of the antenna rather than from the junction of line and antenna.

King and Schmitt² have recorded the reflected wave in an experiment with conditions closely approximating those of Fig. 3. (They used a monopole antenna of length $h = 2.74$ m and of radius $r = 2.38$ mm, a 50-ohm coaxial transmission line, and a symmetric, nonoscillating incident pulse $e^+(t)$ of 3-nsec width and 1-nsec rise.) Their oscilloscope trace shows a function quite similar to the one in Fig. 3.

In the same paper, King and Schmitt introduce the concept of "pulse reflection coefficient." Their presentation of this subject calls for a few comments. They deal with three different coefficients for the reflected pulses appearing on a line of characteristic impedance R_c which drives an antenna of impedance $Z(f)$:

1) an experimental reflection coefficient.

$$2) \quad |\bar{r}| = \left| \frac{\bar{Z}(f) - R_c}{\bar{Z}(f) + R_c} \right|$$

where

$$\bar{Z}(f) = (1/f_c) \int_0^{f_c} Z(f) df,$$

$$3) \quad \bar{r} = \frac{1}{2f_c} \int_{-f_c}^{f_c} \frac{Z(f) - R_c}{Z(f) + R_c} df,$$

¹ C. W. Harrison, Jr., "Tables of Impedance and Radian Effective Half-Length of Electrically Long Cylindrical Antennas of Fixed Length-to-Diameter Ratio," Sandia Corp., Albuquerque, N. Mex., No. SCR-576; November, 1962.

² R. W. P. King and H. J. Schmitt, "The transient response of linear antennas and loops," IRE TRANSACTIONS ON ANTENNAS AND PROPAGATION, vol. AP-10, pp. 222-228; May, 1962.

where f_c is the maximum "significant" frequency in the spectrum of the incident pulse.

(I have taken the liberty of converting their integration on radian frequency ω to integration on cyclic frequency f .)

Although King and Schmitt report numerical values for the experimental reflection coefficient as obtained under several different conditions, they do not define it. That is, it is impossible to say whether the reported numbers relate the peaks of the reflected and incident pulses, their areas, their energies, or some other parameter. It appears likely that they use the ratio of pulse peaks, but it is not so stated.

A pulse reflection coefficient is useless unless it relates in a well-defined way a parameter of the reflected pulse to a parameter of the incident pulse. King and Schmitt nowhere show that $|\bar{r}|$ or $\bar{r}$ provide such a relation, nor do they show any relation among their undefined experimental coefficients, $|\bar{r}|$ and $\bar{r}$. (I use their notation which suggests that $|\bar{r}|$ is the absolute value of $\bar{r}$; such is not the case.) They only show, purely through numerical computation, that these three quantities fortuitously have about the same value under certain conditions.

It turns out that $\bar{r}$ does provide a specific relation between the reflected wave and a certain incident pulse. In (2), if we take $A=1$ and $t=0$, we obtain $\bar{r}$. This coefficient, then, is just the amplitude of the reflected wave at $t=0$ (which may not be the peak) when the incident pulse is $(\sin 2\pi f_c t)/(2\pi f_c t)$ —it is not anything else.

The criticisms above are generally directed toward definitions of coefficients which purportedly relate two quantities without showing exactly what the relation is. More specifically, I would say that there is no definition of pulse reflection coefficient that is suitable for wide application. The best general relation between reflected wave and incident wave is stated in principle by (1). One may proceed from it to the solution for specific cases. We have given here a few illustrations of such solutions.

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Comments on Surface Waves Along Plasma Slabs*

In two papers^{1,2} which appeared recently, Hodara and Cohn examined the fields which can be supported by a plasma slab along which a longitudinal magnetic field is present. Although this work deals

with waves in a gyrotropic medium, the authors have also examined, as a particular case, the solution for an isotropic slab, which occurs when the magnetic field is absent. Some of their conclusions pertaining to this case are, however, incorrect for the reasons shown below.

In their first paper,¹ the authors show that no oscillatory solutions of their determinantal equations (40) and (41) are possible for the isotropic case (Case 1a: $\omega_b=0$). They then conclude that no guided wave propagation can therefore exist along the slab. This conclusion is unwarranted since, in contrast to the authors' statement that trapped (surface) waves can exist only for oscillatory fields within the slab, other nonoscillatory solutions are possible. This latter type possesses fields which decay away from the air-plasma interface in both the free-space (air) region and in the plasma slab (where the field variation is hyperbolic rather than sinusoidal); for these solutions both R_1^2 and R_2^2 are real and positive.

Surface waves with fields which decay away from the air-plasma interface have, in fact, been known for isotropic slabs.³⁻⁵ These waves are only of the TM (E-mode) type and occur whenever the relative dielectric constant of the plasma

$$\epsilon_p = 1 - (\omega_p/\omega)^2$$

is negative, i.e., $\omega_p > \omega$. The properties of these waves have recently been studied extensively,⁵ and it was shown that they may be either of the forward or backward type with interesting power-carrying features.

These surface waves may be strongly excited and thus account for considerable amounts of energy being carried along the slab; they must therefore be considered in any analysis of the fields supported by a plasma layer, such as the one reported in Hodara and Cohn's second paper.² The authors claim in that study that no end-fire ($\phi = \pm 90^\circ$) radiation is possible when a plasma layer covers a metal plate excited by a magnetic line source. When the surface waves discussed above are properly accounted for, however, a considerable amount of power is present at end-fire under appropriate conditions. One must therefore alter the diagrams plotted in Figs. 3, 7(a) and 8(a) of Hodara and Cohn² which show a null at $\phi = \pm 90^\circ$; although this null exists for the space wave as determined by the stationary phase integration, the total field must include the additional surface-wave contributions due to poles in the integral representation for the field. In fact, under certain conditions, these surface-wave contributions occurring at end-fire form the major contribution to the radiation field.

It is also worthwhile to observe that, in addition to obtaining the correct behavior at end-fire, the existence of poles in the complex integration plane also explains the presence of radiation peaks at an oblique angle,

such as the one shown in Fig. 4(a) of Hodara and Cohn.² When ϵ_p is positive rather than negative, improper complex poles are present instead of the surface wave (and other proper complex poles). These improper poles account for leaky waves which, when strongly excited, produce radiation maxima such as the major peak of Fig. 4(a), as well as the minor peaks shown in the same radiation pattern. While the neglect of these complex poles does not introduce any error into the results obtained by Hodara and Cohn (in contrast to the case for surface waves), the complex pole approach furnishes a simple and systematic picture of how the radiation pattern varies as a function of frequency, slab thickness, or plasma electron density. This approach was used in a recent paper⁶ dealing with radiation from an isotropic plasma slab wherein the radiation peaks were shown to be due to leaky waves. A more general study which contains physical considerations involving the energy transport of complex waves in relation to radiation patterns was also published.⁷

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Author's Reply⁸

Three different points are discussed by Tamir and Oliner in connection with the authors' paper. We agree fully on the first point and disagree on the second point; the discrepancy on the third point centers on the selection of the particular approach in solving certain radiation problems; a choice much determined by the researcher's objectives. Let us review the three points in question.

Tamir and Oliner are correct in pointing out that hyperbolic modes should be considered in the first paper¹ if a complete solution is desired. Because of the authors' interest in modes exhibiting spatial oscillation in the plasma slab, this other mode was not discussed. However, the authors' mathematical development is still correct and the hyperbolic modes are readily obtained from (46):

$$(-\epsilon)K_0 - R_1\tau_1 = 0.$$

$$(-\epsilon)K_0 - R_1\tau_1^{-1} = 0.$$

The above solutions have a solution if $-\epsilon > 0$, hence if $\omega < \omega_p$. The authors' statement at the top of page 456, 2nd column, should be corrected to read "no wave propagation with spatial oscillatory variation in the slab takes place." There is still considerable research to be done in connec-

* Received October 15, 1962.
H. Hodara and G. I. Cohn, "Wave propagation in magneto-plasma slabs," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-10, pp. 452-459; July, 1962.

² H. Hodara and G. I. Cohn, "Radiation from a gyro-plasma coated magnetic line source," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-10, pp. 581-593; September, 1962.

³ W. O. Schuman, "Wellen längs homogener Plasmenschichten," S.-B. Akad. Wiss., Math. u. naturw. Abt., vol. 225, pp. 255-261; April, 1948.

⁴ A. A. Oliner and T. Tamir, "Backward waves on isotropic plasma slabs," J. Appl. Phys., vol. 33, pp. 231-233; January, 1962.

⁵ T. Tamir and A. A. Oliner, "The spectrum of electromagnetic waves guided by a plasma layer," Proc. IEEE, vol. 51, pp. 317-332; February, 1963.

⁶ T. Tamir and A. A. Oliner, "The influence of complex waves on the radiation field of a slot-excited plasma layer," IRE TRANS. ON ANTENNAS AND PROPAGATION, vol. AP-10, pp. 55-65; January, 1962.

⁷ T. Tamir and A. A. Oliner, "Guided complex waves: pt. I—Fields at an interface; pt. II—Relation to radiation patterns," Proc. IEE, vol. 110, pp. 310-334; February, 1963.

⁸ Received February 25, 1963.